OCHA-PP-175
April 2001
hep-ph/0104241

4d Gauge Theory and Gravity Generated from 3d Ones at High Energy

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Abstract

Dynamical generation of 4d gauge theories and gravity at low energy from the 3d ones at high energy is studied, based on the fermion condensation mechanism recently proposed by Arkani-Hamed, Cohen and Georgi. For gravity, the two form gravity in 4d is generated from the multiple copy of the 3d Einstein gravities. Since the 3d Einstein gravity is topological having no physical degrees of freedom, there seems to appear no difficulty of ultraviolet divergences in this gravity. In the gauge model, matter fermions are introduced on the lattice following Wilson. Then, the 4d gauge interactions are correctly generated from the 3d theories even in the left-right asymmetric theories of the standard model. In order for this to occur, the Higgs fields as well as the gauge fields of the extra dimension should be generated by the fermion condensates. Therefore, the generation of the 4d standard model from the multiple copy of the 3d ones is quite promising. It is, however, the doubling problem remains in the weak interaction sector in the simple treatment of the chiral fermions.

1 Introduction

In a recent paper, Arkani-Hamed, Cohen and Georgi [1] have proposed an interesting model in which 5d gauge theory is generated dynamically from the 4d asymptotically free gauge theory at high energy. It is by no means new that the extra gauge potential is generated dynamically at low energy as the fermion pair condensates, but the concept that the dynamical generation of the extra (5th) dimension at the same time is completely new.

We will firstly reformulate their model by generating explicitly the D+1 dimensional gauge theory action from the D dimensional one. In the course of the study, generation of the space-like dimension is found to be easier than that of the time-like one.

Next, we propose a model in which 4d gravity is generated at low energy by the fermion pair condensates from the 3d ones at high energy. In this model the 2-form gravity [2, 3] in 4d is generated from the multiplication of the 3d Einstein gravities. As is well known 3d Einstein gravity has no dynamical degrees of freedom (being a topological theory [4]), so that the difficulty of ultraviolet divergences inherent in Einstein gravity does not appear in this gravity model.

Thirdly, we introduce matter fermions into the 3d gauge theory and see how 4d matter couplings are induced. We use the Wilson's method [5] of introducing fermion on the discrete lattice and find that it works naturally well in our generation of 4d theory from 3d ones. Especially, the action added by Wilson can be reproduced, if the fermion condensates corresponding to the Higgs scalars are generated. Although the doubling problem [6] does not occur in the left-right symmetric theories such as QED and QCD, it occurs in the left-right asymmetric QFD. Nevertheless the possibility of generating the 4d standard model from the simple multiplication of the 3d ones at high energy is quite promising. The doubling problem may not be so trouble now, if we modify our simple treatment of the chiral fermions to the more sophisticated ones [7].

2 Dynamical generation of (D+1)d gauge theory from Dd one at high energy

Following the model by Arkani-Hamed et al. [1], we begin with the gauge theory having gauge groups $(G)^N \times (G_s)^N$, the direct product of independent N sets of group $G \times G_s$. For simplicity we can restrict G to $SU(m)$ and G_s to $SU(n_s)$. These groups are ordered linearly with the index $\{1, \frac{3}{2}, 2, \frac{5}{2}, \cdots, n, n+\frac{1}{2}, n+1, \cdots, N-\frac{1}{2}, N\}$. These indices become an extra discrete coordinate later. At integer index n the group $G(n)$ and its gauge fields $A(n)$ are assigned, while at half integer index $n+\frac{1}{2}$ the strong group $G_s(n+\frac{1}{2})$ and its gauge fields $A(n+\frac{1}{2})$ are assigned. There are Weyl fermions $\chi(n, n+\frac{1}{2})$ and $\chi(n+\frac{1}{2}, n+1)$ which connect, respectively, two groups $(G(n), G_s(n+\frac{1}{2}))$, and $(G_s(n+\frac{1}{2}), G(n+1))$. Namely, the representation of $\chi(n, n+\frac{1}{2})$ is non-trivial $(\{m\}, \{\bar{n}_s\})$ only under the adjacent set of groups $(G(n), G_s(n+\frac{1}{2}))$, while that of $\chi(n+\frac{1}{2}, n+1)$ is non-trivial $(\{n_s\}, \{\bar{m}\})$ only under $(G_s(n+\frac{1}{2}), G(n+1))$. If at the energy lower than Λ_s, the coupling constants $g_s(n)$ of the groups $G_s(n)$ at the position $n+\frac{1}{2}(n = 0-(N-1))$ become commonly strong, then the fermion pair condensates $\langle \chi(n, n+\frac{1}{2})\chi(n+\frac{1}{2}, n+1)\rangle$ are formed at the lower energy than a common energy scale Λ_s. This condensate is a singlet under $G_s(n+\frac{1}{2})$, but transforms as $(\{m\}, \{\bar{m}\})$ under the groups $(G(n), G(n+1))$. Therefore, after the condensates are formed, the independently prepared groups $G(n)(n = 1, \cdots, N)$ with labels $n = 1, \cdots, N$ (or at different points $n = 1, \cdots, N$) are unified by the gauge principle.

To clarify this point furthermore let us denote the condensate as

$$\frac{1}{2\pi(f_s)^{D-1}}\langle \chi(n, n+\frac{1}{2})\chi(n+\frac{1}{2}, n+1)\rangle = U(x; n, n+1) = e^{iaA_0(x,n)}, \qquad (1)$$

where $U(x; n, n+1)$ is a $m \times m$ unitary matrix and $A_0(x, n)$ is a $m \times m$ hermitian matrix playing the role of gauge potential (connection) in the direction of the extra dimension. Here, we adopt the convention in which the index $\{0\}$ represents the extra dimension, while $i = 1, \cdots, D$ denote the original D dimensions.

Now in the low energy effective theory the following gauge invariant terms can be generated:

$$\Delta S = (f_s)^{D-2} \int d^D x \sum_{n=1}^{N} tr\left[(D_i U(x; n, n+1))^\dagger \left(D^i U(x; n, n+1)\right)\right], \qquad (2)$$

where D_i is the covariant derivative of the relevant gauge groups $G(n) \times G(n+1)$,

$$D_i = \partial_i + iA_i(x, n)U(x; n, n+1) - iU(x; n, n+1)A_i(x, n+1). \qquad (3)$$

Using Eq.(1) and the expansion in a, we have

$$\begin{aligned} D_i U(x; n, n+1) &= ia\Big(\partial_i A_0(x, n) - \frac{1}{a}\{A_i(x, n+1) - A_i(x, n)\} \\ &+[A_i(x, n)A_0(x, n) - A_0(x, n)A_i(x, n+1)]\Big) + \cdots, \end{aligned} \qquad (4)$$

which can be identified to the field strength of $\{i,0\}$ component, $iaF_{i0}+\cdots$. Now the newly generated action becomes

$$\Delta S = f_s^{D-2}\int d^Dx \sum_{n=1}^{N} a^2 (F_{i0})(F^i{}_0) + \cdots . \tag{5}$$

The generated action is combined with the original action

$$S = -\frac{1}{2(g_D)^2}\int d^Dx \sum_{n=1}^{N} tr\left(F_{ij}(x,n)F^{ij}(x,n)\right), \tag{6}$$

leading to the $D+1$ dimensional gauge theory of the group G:

$$\begin{aligned} S+\Delta S \;\; &= \;\; -\frac{1}{2(g_{D+1})^2}\int d^Dx \;\; a\sum_{n=1}^{N} \;\; tr(F_{MN}(x,n)F^{MN}(x,n)) \\ &\qquad + (\text{higher order terms in } a), \end{aligned} \tag{7}$$

where M and N run from $0,1,\cdots,D$.

For the result (7) to occur, the extra dimension should be "space-like", and the following relations should hold:

$$a = \frac{1}{g_D(f_s)^{(D-2)/2}} \quad \text{and} \quad (g_{D+1})^2 = a(g_D)^2. \tag{8}$$

In order to generate the "time-like" dimension, the sign of the generated action ΔS should be altered. This is usually impossible, since the collective excitations such as $U(x;n,n+1)$ increase usually the energy of the system. It may, however, possible if we can consider the scalar field $U(x;n,n+1)$ to be dilatonic ones, since the kinetic terms of the dilatonic fields can have the opposite sign compared to those of the usual scalar fields.

Now the propagator of the D+1 dimensional gauge boson in the Feynman gauge reads

$$D_{MN}(p,p_0) = \frac{-ig_{MN}}{\sum_{i=1}^{D} p_i p^i - (\frac{2}{a})^2 \sin^2(\frac{ap_0}{2})} + (\text{higher order terms in } a), \tag{9}$$

where p_0 is the momentum of the extra discrete space generated, satisfying $-\frac{\pi}{a} < p_0 < \frac{\pi}{a}$. If $a \to 0$ and $N \to \infty$, then the continuous $D+1$ dimensional gauge theory is obtained, but at finite a, modification from the continuous 4d gauge theory will manifest itself at higher energy around $1/a$ or $g_D(f_s)^{\frac{D-1}{2}}$.

3 Dynamical generation of 4d gravity from the 3d one at high energy

In this section we apply the mechanism of the last section to gravity. There are various ways to formulate Einstein gravity. Among them we adopt here the so called 2-form gravity or the Ashtekar formalism of it in 4d [2], since 3d and 4d gravities are treated similarly in this formalism, based on the gauge principle.

At high energy we start with the sum of N sets of independent 3d gravity actions based on the local Lorentz group $(G_L)^N$. Here, an important point is that the gauge group (G_L) is taken to be 4d local Lorentz group SO(1,3) (or SO(4)) and not to be 3d SO(1, 2) (or SO(3)). To simplify the following discussion, we use $SO(4) = SU(2) \times \overline{SU(2)}$ as this gauge group G_L. Then, we have the action at high energy as

$$S_G = \frac{1}{2(\kappa_3)^2} \sum_{n=1}^{N} \int d^3x \frac{1}{2} \epsilon^{ijk} B_i^{AB}(x,n) R_{jk}^{AB}(x,n). \tag{10}$$

Here, $B_i^{AB}(x,n)$ is the $SO(4)$ gauge fields and R_{jk}^{AB} is the $SO(4)$ field strength (Riemann curvature) of the spin connections ω_i^{AB} (gauge fields) defined by

$$R_{jk}^{AB} = \partial_{[j}\omega_{k]}^{AB} + \omega_{[j}^{AC}\omega_{k]}^{CB}. \tag{11}$$

The 3d gravity coupling κ_3 is related to the 3d Newton constant G_3 as $(\kappa_3)^2 = 8\pi G_3$.

Now we will show that the action (10) is nothing but the $2N$ times multiple of the 3d Einstein gravity. To do this, we have to decompose $SO(4)$ adjoint field F_{AB} into the adjoint fields F^a and $\bar{F}^a$ of $SU(2)$ and $\overline{SU(2)}$, respectively: By using 't Hooft symbols [8] η_{AB}^a and $\bar{\eta}_{AB}^a$, or explicitly we have

$$\begin{aligned} T_{AB} &= \frac{1}{2}\left(\eta_{AB}^a T^a + \bar{\eta}_{AB}^a \bar{T}^a\right), \quad \text{or explicitly} \\ T_{ab} &= \frac{1}{2}\epsilon_{abc}(T^a + \bar{T}^a), T_{0a} = \frac{1}{2}(-T^a + \bar{T}^a). \end{aligned} \tag{12}$$

Our convention is that $A, B, \cdots$ run within 4d indices, while $a, b, \cdots$ run within 3d indices.

Then, we have the following decomposition:

$$S_G = \frac{1}{2(\kappa_3)^2} \sum_{n=1}^{N} \int d^3x \frac{1}{2} \epsilon^{ijk} \left(e_i^a(n) R_{jk}^a(n) + \bar{e}_i^a(n) \bar{R}_{jk}^a(n)\right), \tag{13}$$

where the 3d vierbeins $e_i^a(n)$ and $\bar{e}_i^a(n)$ are, respectively, $SU(2)$ and $\overline{SU(2)}$ decomposition of $B_i^{AB}(n)$.

Two terms in the action without and with "bar", correspond to $SU(2)$ and $\overline{SU(2)}$, respectively. They are independently duplicated in 3d gravity, but will be self-dual (chiral) and anti self-dual (anti chiral) parts of 4d gravity.

So far the vierbeins e_i^a and $\bar{e}_i^a$ and the spin connections ω_i^a and $\bar{\omega}_i^a$ are independent variables. After solving the equation of motion with respect to the spin connections (or eliminating the spin connections by path integration), we have the torsionless conditions:

$$D_i e_j^a - D_j e_i^a = \bar{D}_i \bar{e}_j^a - \bar{D}_j \bar{e}_i^a = 0, \tag{14}$$

which can be solved, giving the ordinary expression of ω_i^a in terms of e_i^a.

Now we arrive at the $2N$ times duplication of 3d Einstein actions:

$$S_G = \frac{1}{2(\kappa_3)^2} \sum_{n=1}^{N} \int d^3x \left(e(n) R(n) + \bar{e}(n) \bar{R}(n) \right). \tag{15}$$

As is shown by Witten [4], this kind of theory can be rewritten to the Chern-Simons action based on the inhomogeneous Lorentz group ISO(3), which is the so called topological field theories without quantum degrees of freedom.

Coming back to the starting gravity action (10) at high energy, we will add the 2 component fermions $\lambda(n, n+\frac{1}{2})$ and $\lambda(n+\frac{1}{2}, n+1)$ transforming as $(\{4\}, \{\bar{n}_s'\})$ and $(\{n_s'\}, \{4\})$ under the groups (G_L, G_s') and (G_s', G_L), respectively. The strong group G_s' is not necessarily the same one G_s chosen to generate the 4d gauge theories.

In the same way as in the previous section, we may have the fermion pair condensates at energy lower than Λ_s' by the strong interaction based on the group $G_s'(n)$. The condensates transform vector like under the groups $G_L(n) \times G_L(n+1)$, and are expressed by $N \times N$ orthogonal matrix, namely,

$$\frac{1}{2\pi f_g^2} \langle \lambda(n, n+\frac{1}{2}) \lambda(n+\frac{1}{2}, n+1) \rangle = O(x; n, n+1) = e^{a\omega_0(x,n)}. \tag{16}$$

Then, similarly as before, we have the following expression,

$$\begin{aligned} & D_i O(x, n, n+1)^{AB} \\ & = a\Big(\partial_i \omega_0^{AB}(x,n) - \frac{1}{a}\{\omega_i^{AB}(x,n+1) - \omega_i^{AB}(x,n)\} \\ & + \left[\omega_i^{AC}(x,n)\omega_0^{CB}(x,n) - \omega_0^{AC}(x,n)\omega_i^{CB}(x,n+1) \right] + \cdots \Big) \\ & = a R_{i0}^{AB}(x,n) + \cdots . \end{aligned} \tag{17}$$

To approach the 4d gravity, introduction of the 4d vierbein is necessary, but so far we have only two sets of 3d vierbeins $e_i^a(n)$ and $\bar{e}_i^a(n)$ at each point n, the degrees of freedom of which are 18 at each point. Under a certain condition, B_i^{AB} can be expressed in terms of the 16 independent 4d vierbeins E_μ^A at each point as follows:

$$B_i^{AB} = B_{0i}^{AB} = \frac{1}{2}(\eta_a^{AB} e_i^a + \bar{\eta}_a^{AB} \bar{e}_i^a) = \frac{1}{2}\epsilon^{ABCD} E_{C0} E_{Di}. \tag{18}$$

The required condition is one of those which we impose on the anti-symmetric fields $B_{\mu\nu}^{AB}$ of the 2-from gravity so that we may introduce vierbeins and derive Einstein gravity from it [2]. The reason why such conditions arise may be understood by the Meissner effect due to the condensation of string fields to which $B_{\mu\nu}^{AB}$ couple as gauge fields [3].

Now the dominant action at low energy induced by the fermion pair condensates can be written as

$$\Delta S_G = -f_g^2 \int d^3x \ a \sum_{n=1}^{N} \frac{1}{4}\epsilon^{ijk}\epsilon_{ABCD} E_i^A(x,n) R_{k0}^{CD} E_j^B(x,n+1) + \cdots. \tag{19}$$

Using the expression (18) in the original action (10), we are lead to

$$\begin{aligned} & S_G + \Delta S_G \\ = \ & \frac{1}{2a(\kappa_3)^2} \int d^3x \ a \sum_{n=1}^{N} \Big(\frac{1}{4}\epsilon^{ijk}\epsilon_{ABCD} E_0^A(x,n) R_{jk}^{CD} E_i^B(x,n) \\ + \ & \{2a(\kappa_3)^2 (f_g)^2\} \frac{1}{4}\epsilon^{ijk}\epsilon_{ABCD} E_i^A(x,n) R_{k0}^{CD} E_j^B(x,n+1) + \cdots \Big). \end{aligned} \tag{20}$$

If the following conditions are satisfied,

$$a = \frac{1}{2(\kappa_3)^2 (f_g)^2} \text{ and } (\kappa_4)^2 = 2a(\kappa_3)^2, \tag{21}$$

then we arrive finally at

$$S_G + \Delta S_G = \frac{1}{2(\kappa_4)^2} \int d^3x \ a \sum_{n=1}^{N} E(x,n) R(x,n) + (\text{higher order terms in a}), \tag{22}$$

where we have the relation between 4d gravity coupling and 4d Newton constant as $(\kappa_4)^2 = 8\pi G_4$. In the gravity case, generation of the extra dimension may occur both space like and time like dimensions, since both signs are permitted for the generated action ΔS_G.

4 Introduction of matter fermions into the gauge model and the generation of 4d standard model from 3d one

In this section we introduce matter fermions into the previous model, especially that of generating 4d gauge theory from 3d ones at high energy. We will understand that the 4d standard model is promising to be generated in this way. In the following we restrict ourselves to the less challenging version of generating one "space-like" dimension of x^3. Accordingly, we will use $\{i, j, k, \cdots\}$ for 3d indices $\{0, 1, 2\}$, and $\{\mu, \nu, \lambda, \cdots\}$ for 4d ones.

Then, our starting action of matter fermions at high energy is

$$S_f = \int d^3x \sum_{n=1}^{N} \Big(2aK\bar{\psi}(x,n)i\gamma^i D_i(A(n))\psi(x,n) - \bar{\psi}(x,n)\psi(x,n) \Big), \tag{23}$$

where we have introduced the hopping parameter K in place of the mass parameter, following Wilson [5]. In 3d, fermions are 2 component, so that we have to replace ψ by a doublet of the chiral components ψ_L and ψ_R, namely $\psi = (\psi_L, \psi_R)^T$. If the chiral partner does not exist, then we consider the partner to be 0. Even without having γ_5 in 3d, 2 component fermions labelled by L and R are eigenstates of the helicity operator $h = \boldsymbol{\sigma} \cdot \mathbf{p}/|\mathbf{p}|$.

More explicitly, the action reads

$$\begin{aligned} S_f = \int d^3x \sum_{n=1}^{N} \Big(\quad & 2aK\psi_L(n)^\dagger i(D_0 - \boldsymbol{\sigma}\mathbf{D})\psi_L(n) \\ & +2aK\psi_R(n)^\dagger i(D_0 + \boldsymbol{\sigma}\mathbf{D})\psi_R(n) \\ & -\psi_R(n)^\dagger\psi_L(n) - \psi_R(n)^\dagger\psi_L(n) \quad \Big). \end{aligned} \tag{24}$$

The action is simply N times multiplication of the 3d fermionic gauge invariant actions of the group $(G)^N$.

After the condensation occurs at the energy lower than Λ_s, the extra gauge field of $A_3(x;n)$ can be generated as

$$U(x; n, n+1) = e^{iaA_3(x,n)}. \tag{25}$$

Then, the additional actions $\Delta_1 S_f$ and $\Delta_2 S_f$ may appear:

$$\Delta_1 S_f \;=\; \int d^3x \sum_{n=1}^{N} K\Big(\bar{\psi}(n)i\gamma^3 U(n, n+1)\psi(n+1)$$

$$
\begin{aligned}
& - \bar{\psi}(n+1) i\gamma^3 U(n,n+1)^\dagger \psi(n)\Big) \\
= & \int d^3x \sum_{n=1}^{N} K\Big(\psi_L(n)^\dagger i(-\sigma^3) U(n,n+1)\psi_L(n+1) + \psi_R(n)^\dagger i\sigma^3 U(n,n+1)\psi_R(n+1) \\
& -\psi_L(n+1)^\dagger i(-\sigma^3) U(n,n+1)^\dagger \psi_L(n) - \psi_R(n+1)^\dagger i\sigma^3 U(n;n+1)^\dagger \psi_R(n)\Big), \qquad (26)
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta_2 S_f &= \int d^3x \sum_{n=1}^{N} K\Big(\bar{\psi}(n) U(n,n+1)\psi(n+1) + \bar{\psi}(n+1) U(n,n+1)^\dagger \psi(n)\Big) \\
&= \int d^3x \sum_{n=1}^{N} K\Big(\psi_R(n)^\dagger U(n,n+1)\psi_L(n+1) + \psi_L(n)^\dagger U(n,n+1)\psi_R(n+1) \\
&\quad +\psi_L(n+1)^\dagger U(n,n+1)^\dagger \psi_R(n) + \psi_R(n+1)^\dagger U(n,n+1)^\dagger \psi_L(n)\Big). \qquad (27)
\end{aligned}
$$

Now, we can understand that the first additional action $\Delta_1 S_f$ leads to the fermion's kinetic term as well as the gauge interaction with A_3, namely

$$
\begin{aligned}
\Delta_1 S_f &= \int d^3x \sum_{n=1}^{N} \Big(2aK \left(\bar{\psi}(n) i\gamma^3 \frac{1}{2a}[\psi(n+1) - \psi(n-1)]\right) \\
&\quad -aK\left(\bar{\psi}(n)\gamma^3 A_3 \psi(n+1) + \bar{\psi}(n+1)\gamma^3 A_3 \psi(n)\right)\Big) + \cdots. \qquad (28)
\end{aligned}
$$

The second additional action $\Delta_2 S_f$ is that Wilson proposed a long time ago. This action is combined with the mass-like term in the original action, giving

$$
\Delta_2 S_f = \int d^3x \sum_{n=1}^{N} \left(K\left(\bar{\psi}(n)\psi(n+1) + \bar{\psi}(n+1)\psi(n)\right) - \bar{\psi}(n)\psi(n)\right). \qquad (29)
$$

Combining these generated actions with the original fermion action, we obtain the 4d gauge invariant fermion action having higher order corrections in a. The propagator of the fermion S_F reads

$$
S_F(\mathbf{p}, p_3) = \frac{i}{\sum_{i=0}^{2} \gamma^i p_i + \gamma_3 \frac{\sin(p_3 a)}{a} - m}, \qquad (30)
$$

having the mass

$$
m = \frac{1 - 2K\cos(p_3 a)}{2aK}. \qquad (31)
$$

If the theory is left-right symmetric, the above reasoning works well, and we have no doubling problem as is shown by Wilson [5]: The particle with momentum $p_3 \sim 0$ has mass $(1-2K)/2aK$, while the doubling partner (having opposite chirality) with $p_3 \sim \pi/a$ has the larger mass of $(1+2K)/2aK$, so that for sufficiently small a, the unwanted partner can be decoupled. Therefore, the left-right symmetric 4d QED and QCD can be reproduced without any troubles.

Next, we try to generate the 4d standard model from 3d ones. The standard mode is a gauge theory based on the group $SU(3)_C \times SU(2)_L \times U(1)_Y$, in which the fermions are left-right asymmetric. Even though the first additional action $\Delta_1 S_f$ can be generated, by reproducing correctly the 4d standard model "gauge interactions", the second additional action $\Delta_2 S_f$ can not be generated properly in the left-right asymmetric part of $SU(2)_L \times U(1)_Y$.

In order to remedy this point, we introduce the other fermion pair condensates $H(x; n, n+1)$, generating the third additional action $\Delta_3 S_f$:

$$\Delta_3 S_f = \Delta_2 S_f (KU \to aH). \tag{32}$$

Namely, we replace the gauge fields by the Higgs fields, and supply the Wilsonian action $\Delta_3 S_f$ in the left-right asymmetric theory.

In the standard model, we have this action as

$$\begin{aligned} \Delta_3 S_f &= \int d^3x \sum_{n=1}^{N} a\Big(q_L(n)^\dagger H(n, n+1) u_R(n+1) \\ &\qquad + q_R(n)^\dagger \tilde{H}(n, n+1) d_R(n+1) + h.c. \Big), \end{aligned} \tag{33}$$

where as usual $H(n, n+1) = (\phi^0, \phi^-)^T$ is the Higgs doublet and $\tilde{H}(n, n+1) = (-\phi^+, \phi^0 *)^T$.

Then, the vacuum expectation value of the Higgs, $\langle \phi^0 \rangle = v$, gives the mass of u and d quarks as

$$m_u = \frac{v \cos(p_3 a)}{K_u}, \quad \text{and} \quad m_d = \frac{v \cos(p_3 a)}{K_d}, \tag{34}$$

and the Yukawa couplings as

$$y_u = \frac{\cos(p_3 a)}{K_u}, \quad \text{and} \quad y_d = \frac{\cos(p_3 a)}{K_d}. \tag{35}$$

The doubling particles with $p_3 \sim 0$ and $p_3 \sim \pi/a$ has the opposite sign in the mass as well as in the Yukawa coupling. These values are, however, made to be identical by a chiral rotation of fermion fields. Therefore the doubling problem between particles with $p_3 \sim 0$ and $p_3 \sim \pi/a$ can not be solved in our simple usage of the Wilson fermions.

In order to solve this problem properly, we need, for example, the recent developments such as the domain wall fermion [7] and so on.

Anyway, in the promising scenario of deriving 4d standard model at low energy from 3d ones at high energy the Higgs field (fundamental representation) as well as an extra component of the gauge fields (adjoint representation) should be generated at low energy as the fermion pair condensates due to the strong interactions based on the group G_s.

Although we have not specify the strong interaction sector explicitly, it may be a kind of the "chiral model". It is because we know well that the scalar condensate of fermions is produced as the σ meson which corresponds to H (Higgs scalar) in our case, while the pseudo scalar condensates of π mesons correspond to our U (an extra component of gauge fields).

Finally we will comment on the fact that the the gauge field $U(x; n, n+1)$ is the connection of the discrete lattice on which the same chirality fields are located. On the other hand, the Higgs fields $H(x; n, n+1)$ can be considered also the "connections" of the discrete lattices on which the opposite chirarity fields are located. This remind us of the treatment by Connes of the Higgs scalar [9] in which the Higgs scalar as the connection of the discrete lattice in the non-commutative geometry. Matter coupling of fermions when the gravity is switched on may be studied similarly.

5 Conclusion

In this paper, by generalizing the work by Arkani-Hamed et al. [1], generation of 4d gauge theories and gravity at low energy from the 3d ones at high energy is studied. In the gauge theories, the generation of the space-like dimension is found to be easier than that of the time-like one. Using the 2-form gravity (or the Ashtekar formalism) [2, 3], the 4d Einstein gravity is shown to be generated at low energy from the $2N$ times multiply of the 3d Einstein gravity which is topological [4] having no physical degrees of freedom. Therefore our gravity theory is safe from the ultraviolet divergences. For the gravity there is no difference between the generation of time-like and space-like dimensions. The fact that the 4d gravity becomes 3d one at high energy is thought to be something to do with the holographic principle [10].

Introduction of matter fermions is studied following Wilson with the so called Wilson fermions [5]. The matter couplings are correctly reproduced when coming down from 3d theory to 4d one. To reproduce the standard model interactions two kinds of fermion condensates should be reproduced. One of them becomes the extra component of the gauge fields and the other becomes the Higgs field at low energy. In this way the left-right asymmetric interactions such as the 4d standard model interactions are possible to be generated at low energy, starting from the multiplicated 3d ones at high energy. However, the doubling problem of having the opposite chiral partner remains in the left-right asymmetric part (weak interactions), but not in the left-right symmetric parts (electromagnetic and strong interactions). The remained difficulty might be solved by modifying the treatment of chiral fermions from our simple one to the more sophisticated ones [7]. In order to give the firm foundation to our models, it should be clarified the dynamics of how the fermion pair condensation occurs so as for the gauge fields, the Higgs fields and the spin connections of gravity to be properly produced.

The most interesting next problem is how these theories are seen in high energy experiments and observations. Probably the scale of the metric's (gravity) generation Λ'_s is sufficiently larger than that of the standard model's generation scale, Λ_s. The Feynman rules of gauge theories and gravity become modified when the lattice constant will manifest itself at high energy. Using these modified rules, we have to clarify the constraints from the available high energy experiments and observations. At the same time we have to show the experimental and observational prospects of our models in the future.

Acknowledgment

The author is grateful to Gi-Chol Cho who introduces this new topics to the author by suggesting the possibility of generating 4d theory from 3d one, having useful discussions and reading the manuscript. He also thanks Etsuko Izumi for her collaboration with him.

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